## Projectile Motion

## Safety Requirements

Safety Glasses (not necessarily chemical goggles) are required.
Safety equipment is required based on the hazards present in the room, not the activities being performed at the time. Whenever students have projectile launchers, this hazard is present and glasses are required.

## Equipment

- Computer with spreadsheet software
- Projectile Launcher
- Ramrod
- Yellow Plastic Ball
- Carbon Paper
- Meter Stick, Measuring Tape, Ruler


## Objectives

- To investigate how the launch angle affects the range of a projectile.


## Introduction

In this experiment, you will study how the range of a projectile changes as the angle at which it is fired is changed. A plastic ball fired from a spring gun is the projectile. The gun is placed on a floor, and the ball is fired into the air at different angles, striking the floor. To help locate the point of impact on the floor, use a carbon paper and a white paper. Ask your instructor for any other simple devices and instruments. Vary the angles by as great a range as possible.

The experiment will proceed in two stages. First, we will use a horizontal shot to establish the launch speed of the ball. Then, we will launch the ball at angles above horizontal to observe the effect on the range.

Our basic assumption is that a flying object has an acceleration of $9.8 \mathrm{~m} / \mathrm{s}^{2}$ downward. The basic formulas of kinematics apply. When we put them together with the initial velocity and height, we can derive Eq. 1. (This comes from the basic Projectile Motion equations, in Sec 4.3 of Serway/Jewett $9^{\text {th }}$ Edition, or Sec 3.4 of Serway/Vuille $10^{\text {th }}$ Edition. You can feel free to solve the basic equations instead of using Eq. 1.)

We will use a comparison between experimentally measured ranges $x$ and the same ranges calculated using Eq. 1 as the criteria for the success of the experiment.

$$
\begin{equation*}
x=v_{0} \cos \theta \frac{v_{0} \sin \theta+\sqrt{2 g h+\left(v_{0} \sin \theta\right)^{2}}}{g} \tag{1}
\end{equation*}
$$

## Experimental Setup

The experimental setup consists of a spring-loaded launcher (Figure 1) and a plastic ball used as a projectile. A sheet of carbon paper placed on the top of a sheet of white paper will record the landing position of the projectile. Tape the white paper to the floor first, then place the carbon paper on top of it, shiny side down. When you want to see where the ball landed, just look at the white paper for a black mark from the carbon.


Figure 1. Projectile launcher set to an angle above horizontal. (Image credit: PASCO)

## Part 1: Horizontal Launch - Initial Velocity of the Projectile

Each launcher has a unique spring that results in different initial velocity of the projectiles. You first need to determine the initial velocity of your launcher. Note: you have to use the same "range" setting on the launcher for every shot. That is an important experimental parameter to make note of and communicate to your readers.

1. Use the plumb line of the launcher to position the barrel horizontally.
2. Measure the height, $h$, from which the projectile is launched.
3. Fire several shots horizontally and measure the resulting ranges, $x_{i}$
4. Calculate the initial speed of the projectile. (Hint: The time it takes the projectile to fall from the launch height to the ground is a good middle step. Alternatively, just set $\theta=0^{\circ}$ in Eq. 1.)

| Height <br> $(\mathrm{m})$ | Range (m) |  |  |  |  |  | Initial <br> Speed <br> $(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Trial 1 | Trial 2 | Trial 3 | Trial 4 | Trial 5 | Average |  |
|  |  |  |  |  |  |  |  |

Table 1. Recorded ranges of the horizontally launched projectile.

## Part 2: Angled Launch

For the angled launches, use the hanging plumb bob to measure the angle of the launcher.

1. Fire the plastic ball at increasing angles, ranging from $10^{\circ}$ to $80^{\circ}$, in increments of $10^{\circ}$. You might want to add an extra data point at $45^{\circ}$.
2. At each angle, do 3 trials. You may need practice shots to figure out where to place the carbon paper. Practice shots don't count as trials unless you can definitely say where the ball landed.
3. For each trial, measure the range from the launcher. The initial horizontal position of the projectile could be found by dropping a vertical line from the starting point marked by a cross on the surface of the launcher.
4. Calculate the expected Range for each angle in Table 2. Excel formula for calculation is provided below where the bold symbols need be replaced by the numerical values. For value of the initial speed of the projectile, use the number from Table 1.
$=\mathbf{V}_{\mathbf{o}}{ }^{*} \cos (\operatorname{radians}(\boldsymbol{\theta}))^{*}\left(\mathbf{V}_{\mathbf{o}}{ }^{*} \sin (\operatorname{radians}(\boldsymbol{\theta}))+\operatorname{sqrt}\left(2^{*} 9.8^{*} \mathbf{h}+\left(\mathbf{V}_{\mathbf{o}}{ }^{*} \sin (\operatorname{radians}(\boldsymbol{\theta}))\right)^{\wedge} 2\right)\right) / 9.8$
5. Plot the two series of values (experimental and calculated) on the same graph. Use scatter plot of Range vs. Angle. (Remember, the *FIRST* variable is the *VERTICAL* axis.)
6. Fit the two series into polynomial function and display equations on the graph.
7. Examine both trends and conclude how the launch angle affects the range; make sure to mention at what launch angle the projectile reaches its maximum range.
8. Comment on the agreement between the experimental and calculated values of the range.

| Angle ( ${ }^{\circ}$ ) | Measured Range (m) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
|  | Trial 1 | Trial 2 | Trial 3 | Average |  |
| 0 |  |  |  |  |  |
| 10 |  |  |  |  |  |
| 20 |  |  |  |  |  |
| 30 |  |  |  |  |  |
| 40 |  |  |  |  |  |
| 45 |  |  |  |  |  |
| 50 |  |  |  |  |  |
| 60 |  |  |  |  |  |
| 70 |  |  |  |  |  |
| 80 |  |  |  |  |  |

Table 2. Recorded data for angled launches of the projectile.

## Requirements for the Report (also consult the rubric):

Save your Excel files through your Blackboard Group File Exchange

- The abstract section must contain the following explanations in paragraph form:
- The relationship between what two quantities was investigated in the experiment.
- How the data was collected for Table 1: the setting of the launcher's angle, the arrangement of carbon and white paper to mark the landing spot of the projectile, the measurements of the initial elevation and the landing range (what tools were used and how they were used).
- How the initial velocity of the projectile was calculated with the statement of the formula used in the calculation.
- How the angle was adjusted from zero to 80 degrees to collect data for Table 2 and how the calculated values of the corresponding ranges were obtained.
- How the data from Table 2 was graphed to analyze the trend in the range (include both, the experimental and calculated series).
- The description of the trend (As the angle increases from zero to ... degrees, the value of the range ....) along with the trendline equation and the statement on the correlation between measured and calculated curves.
- The summary statement on how the variations in angle affected the landing range of the projectile.
- The data section must include
- 2 Tables (labeled and captioned)
- 1 Graph with two series plotted on it (titled, axis labels, units, labeled and captioned)
- Measured landing range
- Calculated landing range

